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Model Reduction of Large Space Structures Using Approximate Component Cost Analysis

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State-space model reduction methods, such as the component cost analysis (CCA), requires the solution of Lyapunov equations of order equal to the order of the system. However, for many engineering applications, the storage requirements and computational time needed for the solution of such large Lyapunov equations is often prohibitive. In this work, the use of Krylov-subspace iterative methods is examined to obtain low-rank approximate solutions of Lyapunov equations for use in CCA model reduction of large space structures. The methods are applied to obtain reduced-order models of the International Space Station multibody assembly stages for simulation and control purposes. In addition, closed-form expressions for cost-equivalent and cost-decoupled realizations are derived based on the approximate Lyapunov solutions. It is shown that using the proposed methods, approximate CCA reduced-order models can be obtained with a significant reduction in the computational effort and time.

I. Introduction

SYSTEM analysis, simulation, and control design objectives often dictate the need to approximate high-order dynamical systems by lower-order ones. A number of model reduction methodologies, such as balanced truncation,[1] Hankel-norm approximation,[2] and optimal L_2 and H_∞ methods,[3–8] have been developed in the past two decades to address this objective. In particular, model reduction plays an indispensable role in the analysis and control of flexible space structures because these systems are by nature distributed parameter, and discretization methods result in high-order dynamic models (see Refs. 9 and 10 and references therein).

A state-space methodology that is particularly suitable for the model reduction of multibody flexible structural systems is component cost analysis (CCA). CCA associates a component output cost with each system state or component. Then, the system components are ranked according to their cost contribution, and reduced-order models are obtained by deleting (truncating) the states with the smallest cost.[11–14] CCA has been applied successfully to obtain reduced-order models of space systems and flexible structures.

In CCA, the computation of the component costs requires the solution of a Lyapunov equation of order equal to the order of the full-order system. However, in many engineering applications, multiple model reductions of very large-scale systems are required for simulation and control design purposes. For example, finite element modeling of large multibody space structures, such as the International Space Station (ISS), could result in systems with several hundreds or thousands of states. Solution of Lyapunov equations of this order is difficult or prohibitive to obtain due to excessive computational time and storage requirements.

In this work, the use of Krylov-subspace iterative methods[15–17] are proposed to obtain low-rank approximate solutions of large Lyapunov equations for approximate CCA model reduction of very large-scale systems. Cost-equivalent and cost-decoupled realization using the approximate Krylov-subspace solutions are developed. The methods are applied to obtain reduced-order models for several ISS assembly stages, and extensive comparisons with the standard methods are provided. It is shown that the proposed approximate CCA methods are computationally efficient and produce close approximations of the exact CCA solutions.

The remainder of this paper is organized as follows. Section II provides a brief overview of the CCA method, and Sec. III introduces the Krylov-subspace solution method that will be used for solving large Lyapunov equations. Section IV presents the proposed approximate CCA method and the derivation of the approximate reduced-order cost-equivalent and cost-decoupled realizations. Section V provides computational results for the ISS assembly stages and extensive comparisons with the exact methods.

II. CCA

Consider a linear, time-invariant (LTI) dynamic system of order n with state-space representation

$$\dot{x} = Ax + Bw, \qquad y = Cx \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^l$ is the system output, and $w(t) \in \mathbb{R}^m$ is the input vector. The model reduction problem consists of finding a low-order LTI state-space model

$$\dot{x}_r = A_r x_r + B_r w, \qquad y = C_r x_r \tag{2}$$

where $x_r(t) \in \mathbb{R}^p$ and $p \ll n$, to approximate system (1).

We assume that $w(t)$ is a zero-mean white noise with intensity $W > 0$. Then, a system performance metric of interest is the output quadratic cost

$$V = \lim_{t\to\infty} \mathcal{E}\{y^T Q y\} \tag{3}$$

where Q is a given weighting matrix and $\mathcal{E}(\,)$ denotes the expectation operator. CCA consists of decomposing V into the sum of contributions V_i associated with each state x_i, where each V_i is defined by

$$V_i = \lim_{t\to\infty} \mathcal{E}\left\{ \frac{\partial\{y^T Q y\}}{\partial x_i} x_i \right\} \tag{4}$$

Note that the component costs are finite if there are no unstable observable modes and they satisfy the cost-decomposition property

$$V = \sum_{i=1}^{n} V_i$$

It can be easily shown[14] that V_i is computed by

$$V_i = [X C^T Q C]_{ii} \tag{5}$$

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where X is the state covariance matrix satisfying the Lyapunov equation

$$XA^T + AX + BWB^T = 0 \tag{6}$$

For model reduction purposes, the component costs are ranked in the following manner:

$$|V_1| \geq |V_2| \geq \cdots \geq |V_n|$$

where, to simplify our bookkeeping, we have assumed that the states have been arranged according to their component costs. Hence, in this case the most critical state of the system is x_1 having component cost V_1, and the least critical state is x_n having component cost V_n. The CCA model reduction scheme consists of simply discarding (truncating) the states with the lowest component cost.[11–13] These states have the smallest contribution in the system output cost V.

In general, CCA can be applied to system components or subsystems where each subsystem is described by a number of state variables. Then, by the attribution of a component cost to each subsystem, an evaluation can be made of its importance in the overall dynamic response. In the special case where the components are modal coordinates, the CCA procedure is called modal cost analysis.[18] CCA of multibody interconnected mechanical systems can be accomplished by including the nonworking constraint forces of the interconnected components in the system output vector. In this way, the system coordinates are ranked according to their combined contribution in the system response and the preservation of the nonworking constraints forces.[19]

III. Approximate Solution of Large Lyapunov Equations

Model reduction using the CCA approach requires the solution of the Lyapunov equation (6) to obtain the system's covariance matrix X. Numerical solutions of the Lyapunov equation has been examined for the past several decades. Early algorithms employed a Kronecker product expansion that transformed the Lyapunov equation into a large, sparse linear system of n^2 equations with n^2 unknowns, requiring $\mathcal{O}(n^6)$ floating-point operations (FLOPs) for numerical solution.[20] More recently, practical solution procedures for small ($n \leq 200$) Lyapunov equations are based on the Bartels–Stewart algorithm[21] or the closely related Hammarling algorithm[22] that make direct use of the matrix structure of the Lyapunov equation. Both methods make use of the real Schur decomposition $A = USU^T$ to transform the Lyapunov equation to a new set of coordinates, where the unknown matrix S is lower block triangular with 2×2 blocks along the diagonal. With this choice of coordinates, the Lyapunov equation can be solved directly for its solution X or for its Cholesky factor $GG^T = X$. Both the Bartels–Stewart algorithm and the Hammerling algorithm require $\mathcal{O}(n^3)$ FLOPs to obtain the solution.

Unfortunately, the methods are not suitable for solving very large Lyapunov equations inasmuch as the computational time and storage requirements are prohibitive. Recently, iterative schemes, such as successive overrelaxation (SOR),[23] alternating direction implicit (ADI),[24] and Krylov-subspace residual minimization schemes[15] have been proposed to address the solution of such large and sparse Lyapunov equations. SOR and ADI methods require information on the spectrum $\lambda(A)$ and provide full-rank estimates of the solution. For systems with lightly damped modes, typical for large flexible space structures, the cited methods often require a large number of iterations to converge. On the other hand, Krylov-subspace methods provide low-rank approximate solutions for the case where $m \ll n$, that is, for the case of a low-rank B matrix in Eq. (6). They are part of a class of iterative methods originally proposed for the solution of large eigenvalue problems, and they have been applied to the solution of approximation problems in various fields.

In this work, we concentrate on the use of Krylov-subspace methods to obtain low-rank approximate solutions of Eq. (6) for efficient CCA model reduction of large flexible structural systems. For these problems, the input matrix B is of low rank due to the limited number of sensors compared to the number of structural modes. In addition, CCA requires the solution of the Lyapunov equation (6) for the sole purpose of ranking the component costs V_i to determine the low-cost states to be truncated. Hence, we expect that a low-rank approximate solution $\hat{X}$ will result in approximate component costs $\hat{V}_i$ that preserve the ranking of the system components.

In the following, the Krylov-subspace technique is reviewed: Let $K = [B \;\; AB \;\; A^2B \cdots A^kB]$, where k is a given positive integer. Then the kth-order Krylov subspace of (A, B) is the km-dimensional space $\mathcal{K} = \mathrm{span}(K)$. An orthonormal basis E_k for the Krylov subspace $\mathcal{K}$ can be computed via the following Arnoldi process[15,17]:

1) Compute $B = Q_1R$ (QR factorization).

2) Use $j = 1, \ldots, m$.

a) Set $E_j = [Q_1 \;\; Q_2 \cdots Q_j]$.

b) Compute

$$\begin{bmatrix} A_{1j} \\ A_{2j} \\ \vdots \\ A_{jj} \end{bmatrix} = E_j^T A Q_j$$

c) Compute

$$Q_{j+1}A_{j+1,j} = AQ_j - \sum_{k=1}^{j} Q_k A_{kj} \quad \text{(QR factorization)}$$

c) End step 2.

Householder reflections may be used to avoid the loss of orthogonality in E_k that often occurs in the Arnoldi process.[16] Krylov-subspace methods for the numerical solution of the Lyapunov equation (6) make use of the Krylov basis E_k to provide low-rank updates $\hat{X} = E_k \Sigma E_k^T$ to compute an approximate solution. The Krylov-subspace iterative algorithm for the approximate numerical solution of the Lyapunov equation (6) is as follow[15]:

1) $R_0 = BWB^T$; $i = 0$; $X_0 = 0$.

2) Although $\|R_i\|$ is too large, take the following steps.

a) Compute an orthogonal basis $E_i \in \mathbb{R}^{n \times k}$ of the Krylov subspace $\mathcal{K}$.

b) Compute the solution $\Sigma_i \in \mathbb{R}^{k \times k}$ of the following reduced-order Lyapunov equation:

$$\left(E_i^T A E_i\right)\Sigma_i + \Sigma_i\left(E_i^T A^T E_i\right) + E_i^T R_i E_i = 0$$

c) $\hat{X}_{i+1} = \hat{X}_i + E_i \Sigma_i E_i^T$; $i = i + 1$.

d) $R_i = -(A\hat{X}_i + \hat{X}_i A^T + Q)$.

3) End step 2.

4) $\hat{X} = \hat{X}_i$.

Hence, the Krylov-subspace algorithm requires the solution of a $k \times k$ Lyapunov equation that is of significantly lower order than the initial $n \times n$ Lyapunov equation (6). The approximate solution $\hat{X}$ has the form $\hat{X} = E_k X_k E_k^T$ and it has low rank:

$$\mathrm{rank}(\hat{X}) = \mathrm{rank}(X_k) \leq k$$

It is easy to show that $\hat{X}$ satisfies the following Galerkin orthogonality condition[25]:

$$E_k^T(A\hat{X} + \hat{X}A^T + BWB^T)E_k = 0 \tag{7}$$

The Krylov-subspace parameter k can be used to tradeoff between solution accuracy and computational effort.

IV. Krylov Approximate CCA

In this section, the Krylov-subspace approximate solution $\hat{X} = E_k X_k E_k^T$ of the Lyapunov equation (6) will be used for approximate CCA model reduction of large-scale systems. Hence, in the approximate CCA model reduction, the approximate component costs $\hat{V}_i$ are computed as

$$\hat{V}_i = \left[E_k X_k E_k^T C^T QC\right]_{ii} \tag{8}$$

and the states with small approximate component costs are deleted. Note that the computation of $\hat{V}_i$ can be accomplished without computing the Krylov-subspace solution $\hat{X}$, minimizing storage requirements. Hence, the proposed Krylov-approximate CCA model reduction algorithm is as follows:

1) Given the full-order model (1) solve the Lyapunov equation (6) using the Krylov-subspace method to obtain X_k and E_k.

2) Compute the approximate component costs $\hat{V}_i$ from Eq. (8) and rank them in decreasing order of magnitude.

3) Delete the desired number of states that have the lowest approximate component cost to obtain a reduced-order model.

The order p of the reduced-order model can be selected based on the ranking and the relative values of the approximate component costs $\hat{V}_i$. The fidelity of the reduced model can be assessed by examining the costs and deleting only those states with small cost magnitudes. A good model approximation corresponds to preserving as much as possible the value of the total system cost. Using this approach, the fidelity of the reduced-order model can be easily traded with the model complexity determined by p.

In the following, some important properties of the proposed Krylov-subspace-based CCA model reduction are examined. The first issue to examine is the derivation of cost-equivalent and cost-decoupled realizations based on the Krylov solution. Recall that a reduced-order realization (2) is called cost equivalent if it has the same total output cost V as the full-order system.[11,14] Hence, in this case the truncated states have no contribution in the output cost. In our case, we are interested in reduced-order approximate cost-equivalent realizations that have total output cost $\hat{V}$, where

$$\hat{V} = \sum_{i=1}^{n} \hat{V}_i = \mathrm{tr}(\hat{X} C^T Q C)$$

The following result shows that if we project the initial system (1) using the Krylov basis, we get such an approximate cost-equivalent realization.

Theorem 1: Let $\hat{X} = E_k X_k E_k^T$ be the Krylov-subspace approximate solution of the Lyapunov equation (6). Then the following kth-order system provides an approximate cost-equivalent realization of the system (1):

$$\dot{x}_r = E_k^T A E_k x_r + E_k^T B \boldsymbol{w}, \qquad y = C E_k x_r \tag{9}$$

Proof: First note that $\hat{X} = E_k X_k E_k^T$ satisfies the Galerkin orthogonality condition (7), which due to the orthogonality of the Krylov basis $E_k (E_k^T E_k = I)$ results in

$$E_k^T A E_k X_k + X_k E_k^T A^T E_k + E_k^T B W B^T E_k = 0$$

Hence, X_k is the state-covariance matrix for the system (9). The corresponding system output cost V_r now is

$$V_r = \mathrm{tr}\big(X_k E_k^T C^T Q C E_k\big) = \mathrm{tr}\big(E_k X_k E_k^T C^T Q C\big)$$

$$= \mathrm{tr}(\hat{X} C^T Q C) = \hat{V}$$

which is the approximate (Krylov–solution-based) cost of the full-order system (1); that is, systems (1) and (9) are approximate cost equivalent. ∎

CCA is a function of the system realization. Often, in model reduction the analyst may be free to choose the system realization to perform the model reduction. Note that, in general in a given realization, two different component costs V_i and V_j are not independent. That is, the ith component cost V_i is influenced by the component j, i.e., truncating the component j affects V_i. However, CCA in certain coordinates that are called cost-decoupled coordinates results in states with independent costs.[11] Specifically, a realization (A, B, C) of system (1) is called a cost-decoupled realization if it has the property that $XC^T QC$ is a diagonal matrix, where X is the solution of the Lyapunov equation (6). Here, we are interested in deriving approximate reduced-order cost-decoupled realization based on the Krylov-subspace solution $\hat{X}$. These realizations are cost decoupled; that is, they have the benefits that provide independent component costs, but are approximate because they are computed using the Krylov-subspace solution rather an exact Lyapunov solution. The motivation is to facilitate and enable the calculation of cost-decoupled realizations for large-scale systems. Note that standard cost-decoupled transformations[11,14] cannot be applied in our case because they require invertibility of the covariance matrix, and our Krylov approximate solution $\hat{X}$ is singular. The following result provides a family of reduced-order approximate cost-decoupled realizations based on the approximate solution $\hat{X}$.

Theorem 2: A family of reduced-order approximate cost-decoupled coordinates for the system (1) based on the Krylov-subspace solution $\hat{X} = E_k X_k E_k^T$ is computed as follows:

1) Compute the singular value decomposition (SVD)

$$X_k = [U_{11} \quad U_{12}] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{11}^T \\ U_{12}^T \end{bmatrix} \tag{10}$$

2) Compute the SVD

$$\Sigma_1^{\frac{1}{2}} U_{11}^T E_k^T C^T Q C E_k U_{11} \Sigma_1^{\frac{1}{2}} = [U_{21} \quad U_{22}] \begin{bmatrix} \Sigma_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{21}^T \\ U_{22}^T \end{bmatrix}$$

3) Set

$$T_L = \Lambda^{-1} U_{21}^T \Sigma_1^{-\frac{1}{2}} U_{11}^T E_k^T \tag{11}$$

and

$$T_R = E_k U_{11} \Sigma_1^{\frac{1}{2}} U_{21} \Lambda \tag{12}$$

where Λ is an arbitrary nonsingular diagonal matrix. Then, a reduced-order approximate cost-decoupled realization is given by

$$A_d = T_L A T_R, \qquad B_d = T_L B, \qquad C_d = C T_R \tag{13}$$

and the system covariance matrix in these coordinates is

$$X_d = T_L \hat{X} T_L^T \tag{14}$$

Proof: Using the SVD orthogonality conditions, the orthogonality of the Krylov basis E_k, and the definitions (11) and (12), it can be shown that Eq. (14) results in

$$X_d = \Lambda^{-2} \tag{15}$$

In addition, Eq. (13) results in

$$C_d^T Q C_d = \Lambda^2 \Sigma_2$$

Hence,

$$X_d C_d^T Q C_d = \Sigma_2$$

i.e., the cost-decoupled condition is satisfied. The next step is to show that X_d is indeed the covariance matrix of the realization (13); that is, we have to show that the Lyapunov equation

$$A_d X_d + X_d A_d^T + B_d W B_d^T = 0 \tag{16}$$

is satisfied. Note that, using Eqs. (13) and (15), the left-hand side of Eq. (16) results in

$$U_{21}^T \Big[\Sigma_1^{-\frac{1}{2}} U_{11}^T E_k^T A E_k V_k U_{11} \Sigma_1^{\frac{1}{2}} + \Sigma_1^{\frac{1}{2}} U_{11}^T V_k^T A^T E_k U_{11} \Sigma_1^{-\frac{1}{2}}$$
$$+ \Sigma_1^{-\frac{1}{2}} U_{11}^T E_k^T B W B^T E_k U_{11} \Sigma_1^{-\frac{1}{2}}\Big] U_{21} \tag{17}$$

Now consider the Galerkin orthogonality condition (7) for the Krylov-approximate solution $\hat{X}$,

$$E_k^T (A\hat{X} + \hat{X} A^T + B W B^T) E_k = 0$$

and note that due to the orthogonality of E_k and Eq. (10) we obtain

$$E_k^T A E_k U_{11} \Sigma_1 U_{11}^T + U_{11} \Sigma_1 U_{11}^T E_k^T A^T E_k + E_k^T B W B^T E_k = 0 \tag{18}$$

After some algebraic manipulations, the left-hand side of Eq. (18) provides the term inside the brackets in Eq. (17); that is, the Lyapunov equation (16) is verified. ∎

It is noted that the cost decoupled realization has at most l nonzero component costs because rank $(C^T QC) \leq l$. The proposed family of approximate cost-decoupled coordinates (13) is of order l; that is, the components that correspond to zero cost have already been removed.

V. Computational Results

A. Comparison of Lyapunov Solvers

The first set of numerical experiments is used to validate the efficiency of the proposed Krylov-subspace methods compared to the standard Bartels–Stewart algorithm[21] for the solution of the Lyapunov equation

$$AX + XA^T + Q = 0$$

The methods will be compared in terms of computational accuracy that will be measured with the following index:

$$E_x = \frac{\|AX + XA + Q\|_F}{\|A\|_F}$$

where $\| \|_F$ is the Frobenius norm of a matrix. The computational efficiency of each algorithm will be quantified by the number of FLOPs needed to obtain the solution. The algorithms described in Sec. III are used to obtain the Krylov-subspace solution. The tests are performed in comparison with the MATLAB™ 5.2 built-in Lyapunov equation solver, which uses a routine based on the Bartels–Stewart algorithm.[21] This routine, furthermore, is used to solve the reduced-order Lyapunov equation in step 2c of the Krylov-subspace algorithm.

The two methods are tested to find the covariance matrix for several structural models of different sizes, which correspond to different configurations of the ISS assembly phases. These ISS models represent medium size (≥ 200 states) to large size (≥ 600 states) structural systems composed of interconnected dynamic components such as the core body, solar panels, radiators, and antennas. In particular, the following ISS assembly models are used for the comparison: 1R, 3A, 4A, 5A, 8A + OBS and 12A. The corresponding number of degrees of freedom for these systems are: 157, 156, 186, 247, 368, and 897, respectively. A complete description of the configuration of these structural models is provided in Ref. 26.

A summary of the performance of the two algorithms is shown in Tables 1 and 2. Note that, as expected, the Bartels–Stewart solution is significantly more accurate; however, the corresponding computational effort is approximately one order of magnitude higher. In fact, the Bartels–Stewart methods failed to provide a solution for the 1794-state model that corresponds to the ISS assembly model 12A.

B. Approximate CCA of ISS Assembly Stages

In this section, the proposed approximate CCA methods are utilized for model reduction of several structural system cases that correspond to ISS assembly stages. Our objective is to show that the approximate CCA, based on the Krylov-subspace algorithm, results in quite similar ordering of the system's cost decomposition providing accurate reduced-order models.

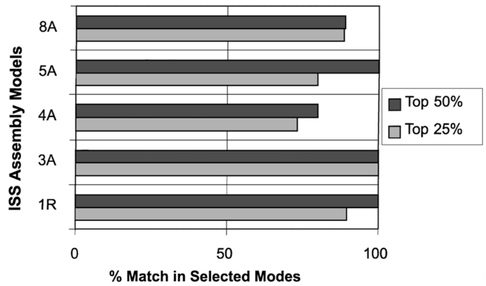

Fig. 1 Mode selection comparison.

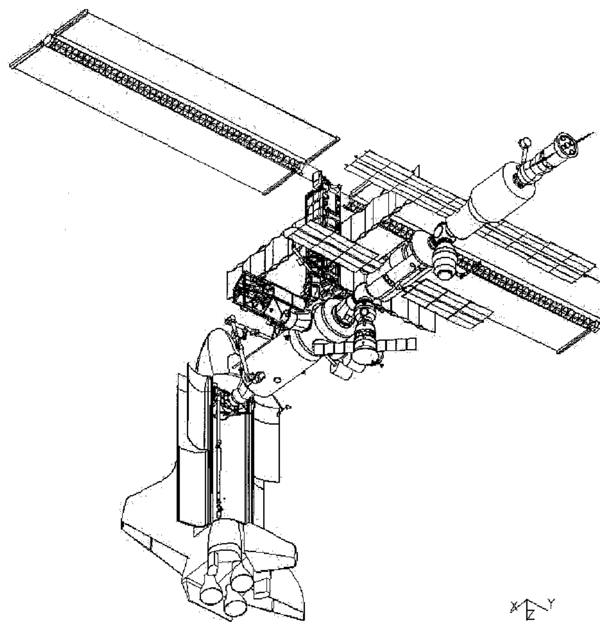

Fig. 2 Isometric view of the ISS assembly stage 8A + OBS.

The first comparison of interest is the comparison of the modes that are preserved as high-cost modes in the CCA using a Bartels–Stewart Lyapunov solver and the approximate CCA that utilizes the Krylov-subspace algorithm. Figure 1 shows the percentage of modes that are common in the two approaches if we seek to preserve the 25% and the 50% of the higher cost modes. The comparison is for the ISS assembly stage models 1R, 3A, 4A, 5A, and 8A. Note that a very large percentage of the high-cost modes are common in the two methods; that is, the two methods will result in quite similar truncated models. In particular, if we are interested in reducing the size of the models by 50%, then both the Krylov-subspace and the Bartels–Stewart Lyapunov solutions will result in CCAs that match over 80% of the higher cost modes.

The second set of experiments consists of a transfer function comparison of the full- and the reduced-order models obtained from the approximate CCA and the exact (Bartels–Stewart-based) CCA model reduction. We consider first the ISS model stage 8A + OBS (see Fig. 2) with 368 modes (736 states) that we seek to reduced by approximatelly 50%. Figures 3 and 4 show the frequency-response comparison of the full- and the reduced-order models using the Bartels–Stewart[21] and the Krylov-subspace solutions, respectively. Note that the two methods result in very similar reduced-order models. A similar comparison is performed for the 12A stage (see Fig. 5), which has 897 modes (1794 states). The frequency-response comparison of the full- and the Krylov-based CCA reduced-order model is shown in Fig. 6. However, note that in this case no Bartels–Stewart Lyapunov solution was possible to obtain for comparison, due to extreme computational requirements of solving a 1794-order

Table 1 Accuracy comparisons of the Bartels–Stewart and the Krylov-subspace methods

ISS assembly stage	Bartels–Stewart method	Krylov-subspace method
1R	$4.3e{-}11$	$5.3e{-}7$
3A	$8.2e{-}11$	$2.8e{-}7$
4A	$1.5e{-}11$	$1.2e{-}6$
5A	$1.3e{-}10$	$1.7e{-}6$
8A + OBS	$9.5e{-}13$	$1.2e{-}6$
12A	Failed	$3.0e{-}6$

Table 2 Computational effort comparisons of the Bartels–Stewart and the Krylov-subspace methods

ISS assembly stage	Bartels–Stewart method	Krylov-subspace method
1R	$5.8e{+}8$	$7.6e{+}7$
3A	$2.1e{+}9$	$4.3e{+}8$
4A	$6.6e{+}9$	$7.2e{+}8$
5A	$9.4e{+}9$	$1.1e{+}9$
8A + OBS	$6.8e{+}10$	$5.5e{+}9$
12A	Failed	$1.1e{+}11$

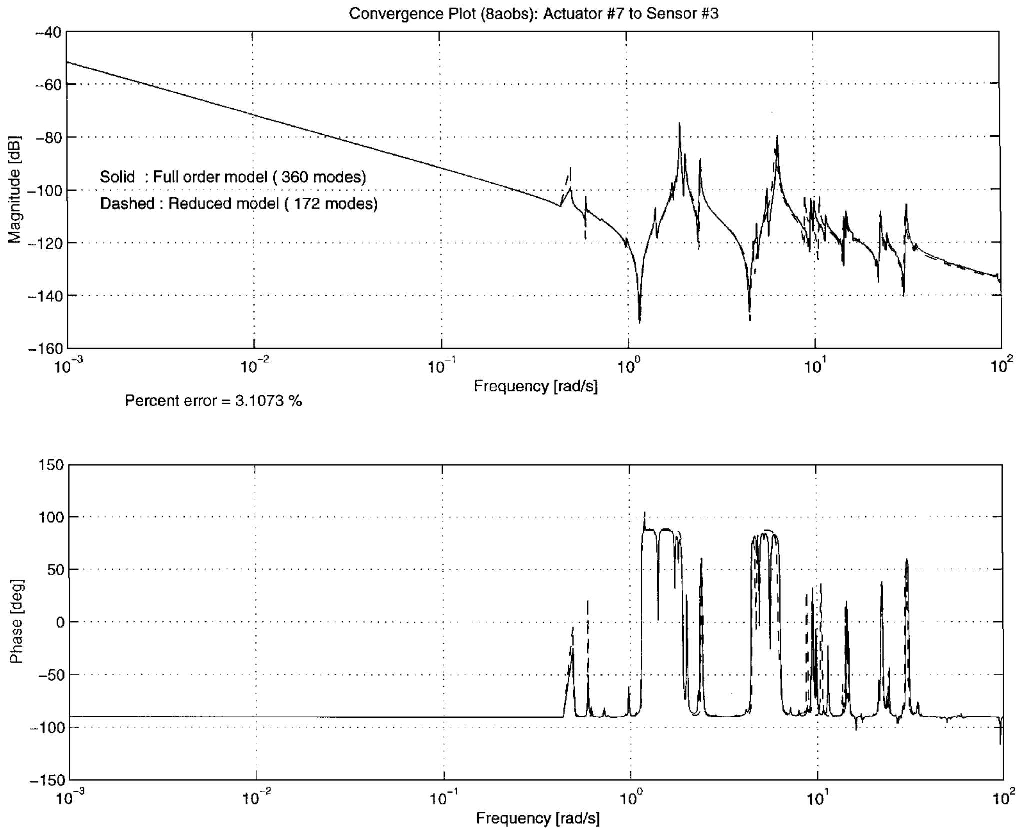

Fig. 3 Frequency-response comparison of the full- and the Bartels–Stewart CCA reduced-order models for 8A-OBS.

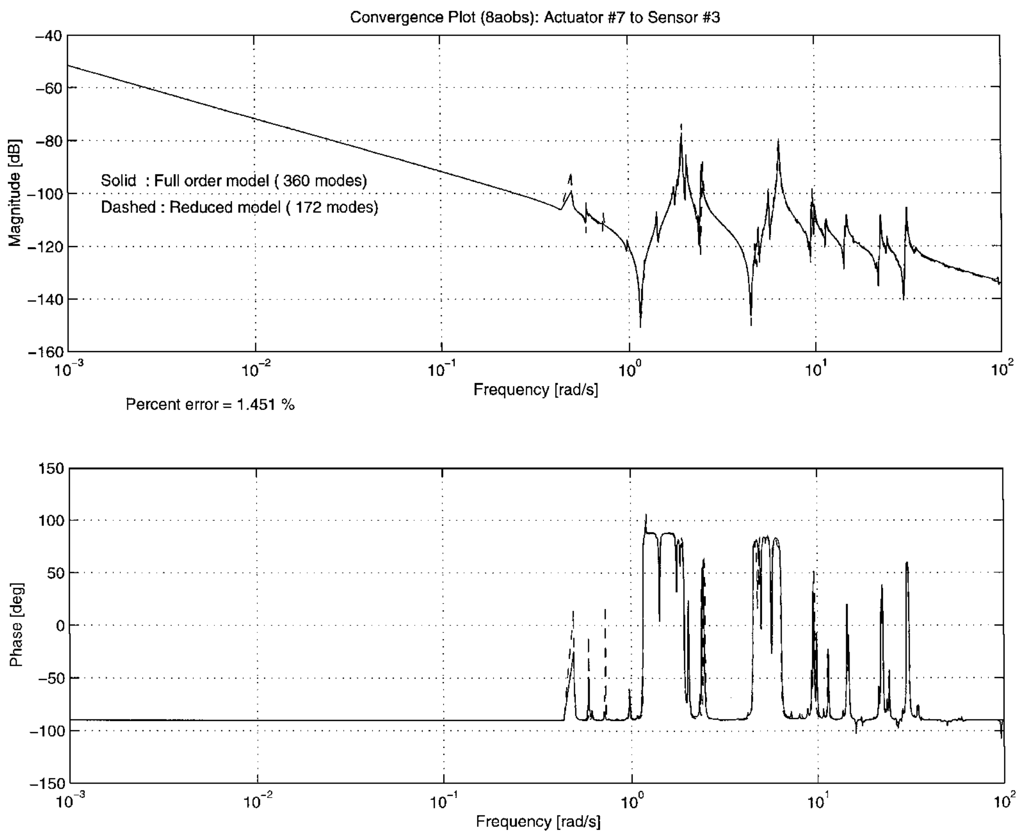

Fig. 4 Frequency-response comparison of the full- and the Krylov-subspace CCA reduced-order models for 8A-OBS.

Lyapunov equation using the Bartels–Stewart method.[21] It is noted that ISS assembly stages beyond the 12A stage correspond to structural models with several thousands of states, and model reductions using the proposed Krylov-subspace solutions could be vital for treating these systems.

VI. Approximate Cost-Decoupled Coordinate

In this section, a low-order structural model example is presented to demonstrate the computation of the proposed approximate cost-decoupled coordinates presented in Sec. IV. To this end, consider the following fifth-order system:

$$A = \begin{bmatrix} -0.0297 & -1 & 0 & 0.0438 & 0 \\ 0.3458 & -0.8927 & -0.0980 & -0.0632 & -0.3396 \\ -1.0837 & -0.2519 & -2.5419 & -1.6431 & -0.9347 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0.3810 & 0.0400 \\ 0.0670 & 1.5900 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1.5 & 2.3 & 0 & 4.1 & 5 \\ 0 & 0 & 2 & 0 & 5 \end{bmatrix}$$

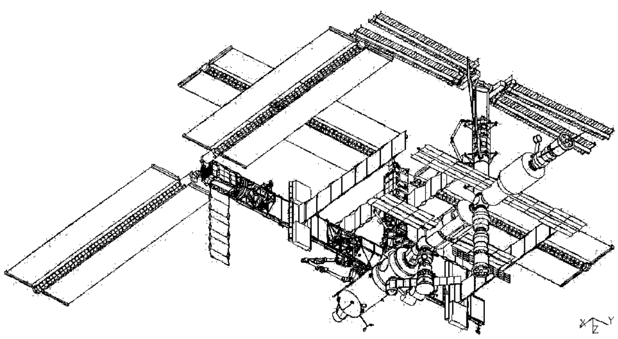

Fig. 5 Isometric view of the ISS assembly stage 12A.

The exact covariance matrix for this system (computed via the Bartels–Stewart algorithm[21]) is

$$X = \begin{bmatrix} 0.1165 & -0.0029 & -0.0195 & 0.0116 & -0.1085 \\ -0.0029 & 0.0807 & 0.0071 & -0.0066 & 0 \\ -0.0195 & 0.0071 & 0.5034 & 0 & 0.0066 \\ 0.0116 & -0.0066 & 0 & 0.3033 & -0.0062 \\ -0.1085 & 0 & 0.0066 & -0.0062 & -0.1265 \end{bmatrix}$$

and the Krylov-subspace approximate solution of order four is

$$\hat{X} = \begin{bmatrix} 0.1149 & -0.0015 & -0.0143 & 0.0080 & -0.1149 \\ -0.0015 & 0.0808 & 0.0067 & -0.0078 & 0.0015 \\ -0.0143 & 0.0067 & 0.4983 & 0 & 0.0143 \\ 0.0080 & -0.0078 & 0 & 0.3038 & -0.0080 \\ -0.1149 & 0.0015 & 0.0143 & -0.0080 & -0.1149 \end{bmatrix}$$

Note that $\mathrm{rank}(\hat{X}) = 4$. The cost-decoupled realization[11,14] of the system computed using the exact covariance X is as follows:

$$A_d = \begin{bmatrix} -0.2406 & -0.1865 & -0.1952 & -0.1824 & 0.3708 \\ 0.9788 & -0.8993 & 2.0117 & -0.2403 & 0.5099 \\ -0.5443 & -0.0555 & -1.1202 & -0.1492 & -0.5363 \\ 0.4580 & 0.0148 & -0.0669 & -0.1318 & 0.0499 \\ -1.2909 & 0.5687 & -.2257 & 0.6791 & -1.0725 \end{bmatrix}$$

$$B_d = \begin{bmatrix} -0.2722 & -0.6380 \\ -0.1982 & 1.3264 \\ 0.5478 & -1.3929 \\ 0.4549 & 0.2380 \\ -1.0916 & -0.9763 \end{bmatrix}$$

$$C_d = \begin{bmatrix} -2.4215 & -1.0953 & 0 & 0 & 0 \\ -1.6574 & 1.6003 & 0 & 0 & 0 \end{bmatrix}$$

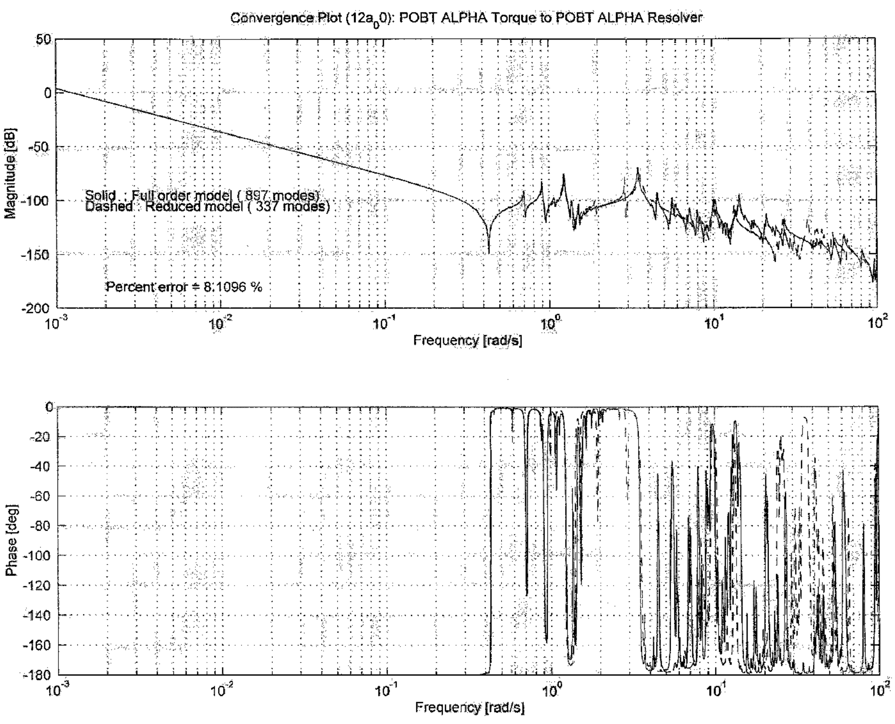

Fig. 6 Frequency-response comparison of the full- and the Krylov-subspace CCA reduced-order models for 12A.

and he output cost matrix in these cost-decoupled coordinates is

$$X_d C_d^T C_d = \begin{bmatrix} 8.6106 & 0 & 0 & 0 & 0 \\ 0 & 3.7606 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which implies that states 3, 4, and 5 should be deleted. Hence, the exact cost-decoupled reduced-order model is as follows:

$$A_{dr} = \begin{bmatrix} -0.2406 & -0.1865 \\ 0.9788 & -0.8993 \end{bmatrix}, \qquad B_{dr} = \begin{bmatrix} -0.2722 & -0.6380 \\ -0.1982 & 1.3264 \end{bmatrix}$$

$$C_{dr} = \begin{bmatrix} -2.4215 & -1.0953 \\ -1.6574 & 1.6003 \end{bmatrix}$$

Using the results of Sec. IV, the approximate reduced-order, cost-decoupled model using the Krylov-subspace solution $\hat{X}$ is obtained as follows:

$$\hat{A}_d = \begin{bmatrix} -0.2662 & -0.2060 \\ 1.0362 & -0.8870 \end{bmatrix}, \qquad \hat{B}_d = \begin{bmatrix} -0.2809 & -0.6735 \\ -0.2036 & 1.3163 \end{bmatrix}$$

$$\hat{C}_d = \begin{bmatrix} -2.3127 & -1.1134 \\ -1.6251 & 1.5844 \end{bmatrix}$$

Note that the approximate reduced-order cost-decoupled realization $(\hat{A}_d, \hat{B}_d, \hat{C}_d)$ is indeed very close to the exact one (A_{dr}, B_{dr}, C_{dr}).

VII. Conclusions

The use of Krylov-subspace algorithms has been examined for approximate CCA model reduction purposes. The proposed algorithms could be valuable to address model reduction of very large-scale systems where common Lyapunov equation solvers, such as the Bartels–Stewart method, fail due to computational and storage requirements. The computations of approximate cost-equivalent and cost-decoupled realizations using the proposed Krylov-subspace solutions have been provided. The approximate CCA model reduction methodology has been compared with the standard method that uses a Bartels–Stewart Lyapunov solver. Model reductions for several structural systems that correspond to ISS assembly stages have been examined, which validate the efficiency and accuracy of the proposed approximate CCA approach.

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References

[1]Moore, B. C., "Principal Component Analysis in Linear Systems: Controlability, Observability and Model Reduction," *IEEE Transactions on Automatic Control*, Vol. 26, No. 1, 1981, pp. 17–32.

[2]Glover, K., "All Optimal Hankel Norm Approximations of Linear Multivariable Systems and Their L_∞ Error Bounds," *International Journal of Control*, Vol. 39, No. 6, 1984, pp. 1115–1193.

[3]Wilson, D., "Model Reduction for Multivariable Systems," *International Journal of Control*, Vol. 20, No. 1, 1974, pp. 57–64.

[4]Zigic, D., Watson, L. T., Collins, E. G., Jr., and Bernstein, D. S., "Homotopy Methods for Solving the Optimal Projection Equations for the H_2 Reduced Order Model Problem," *International Journal of Control*, Vol. 56, No. 1, 1992, pp. 173–191.

[5]Haddad, W., and Bernstein, D., "Combined L_2/H_∞ Model Reduction," *International Journal of Control*, Vol. 49, No. 5, 1989, pp. 1523–1535.

[6]Collins, E. G., Jr., Ying, S. S., Hadad, W. M., and Richter, S., "An Efficient, Numerically Robust Homotopy Algorithm for H_2 Model Reduction Using Optimal Projection Equations," *Mathematical Modeling of Systems*, Vol. 2, No. 2, 1996, pp. 101–133.

[7]Grigoriadis, K. M., "Optimal H_∞ Model Reduction via Linear Matrix Inequalities," *Systems and Control Letters*, Vol. 26, No. 3, 1995, pp. 321–333.

[8]Grigoriadis, K. M., "L_2 and $L_2 - L_\infty$ Model Reduction via Linear Matrix Inequalities," *International Journal of Control*, Vol. 68, No. 3, 1997, pp. 485–498.

[9]Gawronski, W., and Williams, T., "Model Reduction for Flexible Space Structures,"*Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 1, 1991, pp. 68–76.

[10]Gawronski, W., *Balanced Control of Flexible Structures*, Lectures Notes in Control and Information Sciences, Springer–Verlag, London, 1996, pp. 45–106.

[11]Skelton, R. E., "Cost Decomposition of Linear Systems with Application to Model Reduction," *International Journal of Control*, Vol. 32, No. 6, 1980, pp. 1031–1055.

[12]Skelton, R. E., and Yousuff, A., "Component Cost Analysis of Large Scale Systems," *International Journal of Control*, Vol. 37, No. 2, 1983, pp. 285–304.

[13]Yousuff, A., and Skelton, R. E., "Controller Reduction by Component Cost Analysis," *IEEE Transactions on Automatic Control.*, Vol. 29, No. 6, 1984, pp. 520–530.

[14]Skelton, R. E., *Dynamic Systems Control: Linear Systems Analysis and Synthesis*, Wiley, New York, 1988, pp. 240–299.

[15]Hodel, A. S., and Hung, S. T., "Solution and Applications of the Lyapunov Equation for Control Systems," *IEEE Transactions on Industrial Electronics*, Vol. 39, No. 3, 1992, pp. 194–202.

[16]Hodel, A. S., Tenison, B., and Poola, K. R., "Numerical Solution of the Lyapunov Equation by Approximate Power Iteration," *Linear Algebra and its Applications*, Vol. 236, No. 2, 1996, pp. 205–230.

[17]Jaimoukha, I. M., and Kasenally, E. M., "Krylov Subspace Methods for Solving Large Lyapunov Equation," *SIAM Journal on Numerical Analysis*, Vol. 31, No. 1, 1994, pp. 227–251.

[18]Skelton, R. E., Hughes, P. C., and Hablani, H. B., "Order Reduction for Models of Space Structures Using Modal Cost Analysis," *Journal of Guidance and Control*, Vol. 5, No. 4, 1982, pp. 351–357.

[19]Skelton, R. E., Singh, R., and Ramakrishnan, J., "Component Model Reduction by Component Cost Analysis," *Advances in Aeronautics and Astronautics*, edited by J. L. Junkins, AIAA, Washington, DC, 1990, Chap. 16.

[20]Barnett, S., and Storey, C., "Solution of the Liapunov Matrix Equation," *Electronics Letters*, Vol. 2, No. 8, 1966, pp. 466, 467.

[21]Bartels, R. H., and Stewart, G. W., "Solution of the Matrix Equation $AX + XB = C$," *Communications of the Association for Computer Machinery*, Vol. 15, No. 2, 1972, pp. 820–826.

[22]Hammarling, S., "Numerical Solution of the Stable, Non-Negative Definite Lyapunov Equation," *IMA Journal of Numerical Analysis*, Vol. 2, No. 3, 1982, pp. 303–323.

[23]Starke, G., "SOR-Like Methods for Lyapunov Matrix Equations," *Iterative Methods in Linear Algebra*, edited by R. Beauwens and P. de Groen, North–Holland, Amsterdam, 1992, pp. 233–240.

[24]Lu, A., and Wachspress, E. L., "Solution of Lyapunov Equations by ADI Iteration," *Computers and Mathematics with Applications*, Vol. 21, No. 9, 1991, pp. 43–58.

[25]Saad, Y., "Numerical Solution of Large Lyapunov Equations," *Signal Processing, Scattering and Operator Theory, and Numerical Methods*, edited by M. A. Kaashoek, J. H. van Schuppen, and A. C. M. Ran, Birkhaüser, Boston, Vol. 3, 1990, pp. 503–511.

[26]"ISS On-Orbit Assembly, Modeling and Mass Properties," JSC-26557, revision H, Lockheed Martin, Houston, TX, Oct. 1997.